Teaching Through Problem Solving

Session 3

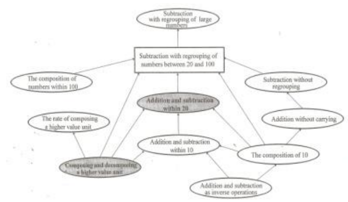
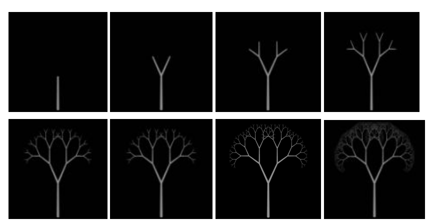


FIG. 1.2. A *knowledge package* for subtraction with regrouping.



facilitated by
Kathy Kubota-Zarivnij

GAINS
Growing Accessible Interactive Networked Supports

- use curriculum expectations and mathematical processes (i.e., grades 6 to 10) to anticipate a range of solutions to a problem, to understand and describe the mathematics in student solutions, and to judge the appropriateness of problems for teaching/learning
- develop a knowledge package or landscape of clustered curriculum expectations
- use the three-part problem solving lesson design to frame the use of problems for teaching mathematics
- develop strategies and mathematics knowledge for anticipating student responses and understanding students' mathematical thinking
- experience strategies for developing students' mathematical communication through the discourse of a math-talk learning community, teacher recording strategies (blackboard writing – mathematical annotations), and coordination and recording of discussion (bansho)
- vary the structure of the problem for students to practise new learning and to provoke use of some strategies and not others, moving towards strategies that can generalize

Looking Back to Look Forward

Multiplication Problem, Bus Problem

- Your goal for learning mathematics *for* teaching
- Characteristic of teaching through problem solving – e.g., content coherence, making connections, nature of learning, nature of student work
- Teacher action – teaching through problem solving

Work of Mathematics Teaching

- Sequencing of math content and curriculum materials
- Generating and using strategic examples and multiple representations
- Talking mathematics and having students talk
- Understanding and analyzing multiple solutions
- Evaluating the mathematical significance of students’ comments and coordinating discussion for learning
- Building correspondence between mathematical ideas, models, and symbols

KKZ, 2008

GAINS

Learn Mathematics *for* Teaching

(Ball, 2005)

- Figure out why procedures work, not just how to do them
- Try to solve problems in more than one way
- Listen to and probe others' thinking, especially when struggling
- Study students' thinking and work
- Talk in class; practise speaking mathematics

Looking Back to Look Forward
- Teaching Through Problem Solving

- Lesson components - Before? During? After?
- Math Content coherence?
- Making Mathematical Connections?
- Who does the Mathematics Work?
- Nature of Mathematics Learning?
- Nature of the Mathematical Work?
- Kind of Mathematics Work Expected?
- Content Elaboration by Whom?

- Your goal for learning mathematics *for* teaching
- Characteristic of teaching through problem solving – e.g., content coherence, making connections, nature of learning, nature of student work
- Teacher action – teaching through problem solving

GAINS

Mathematics Knowledge Package, Landscape, Instructional Trajectory

- Liping Ma (1999) – knowledge package
- Cathy Fosnot (2002, 2007) – landscape of learning
- Marty Simon and Ron Tzur (2003) – hypothetical learning trajectories and key developmental understandings (KDU)
- Set of mathematics knowledge (big idea or concepts, strategies, models of representation) that are organized like a "concept map" to show mathematical connections among learning goals and to map out possible learning paths

Mathematics Knowledge Package (Ma, 1999)

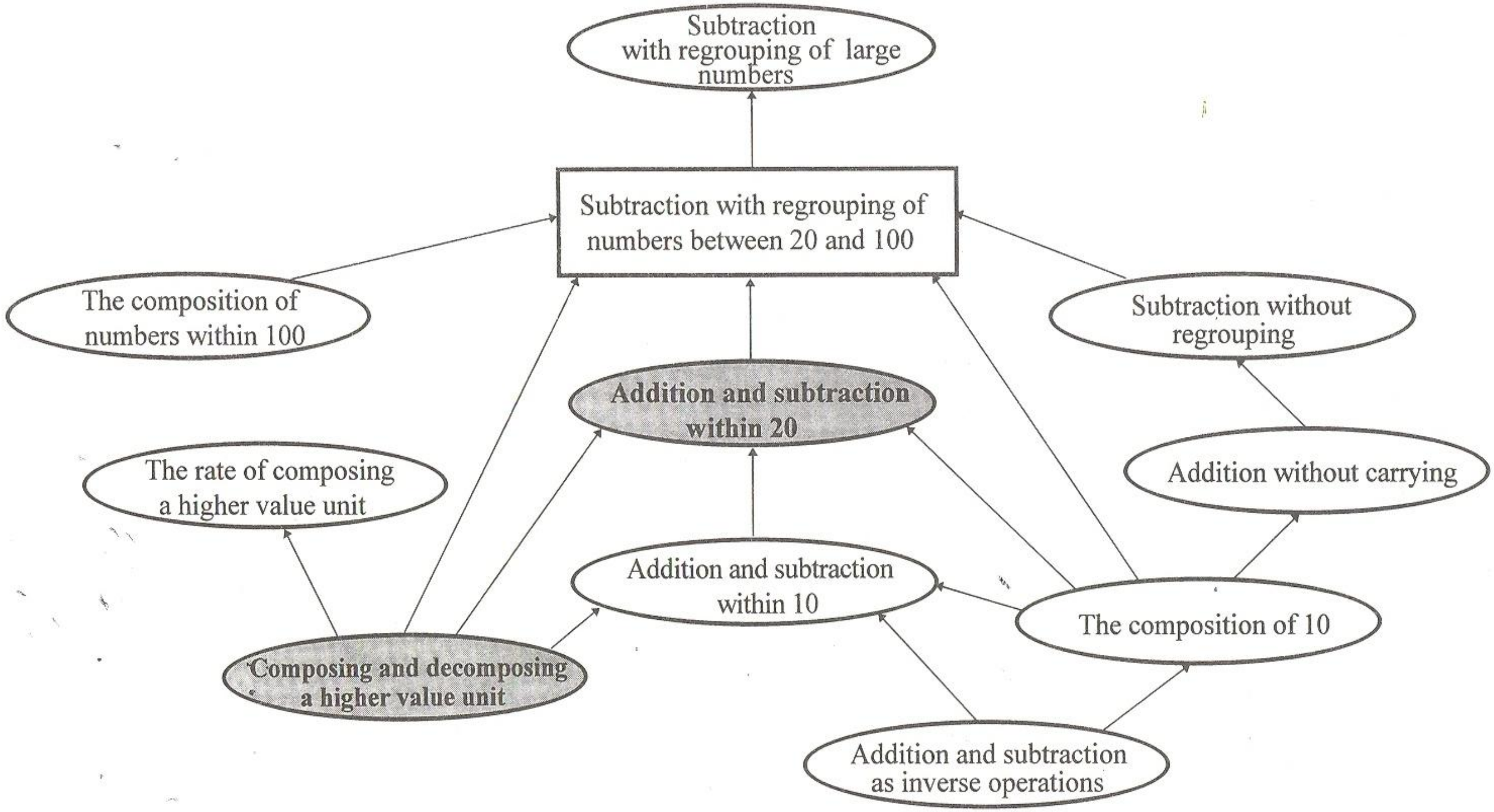
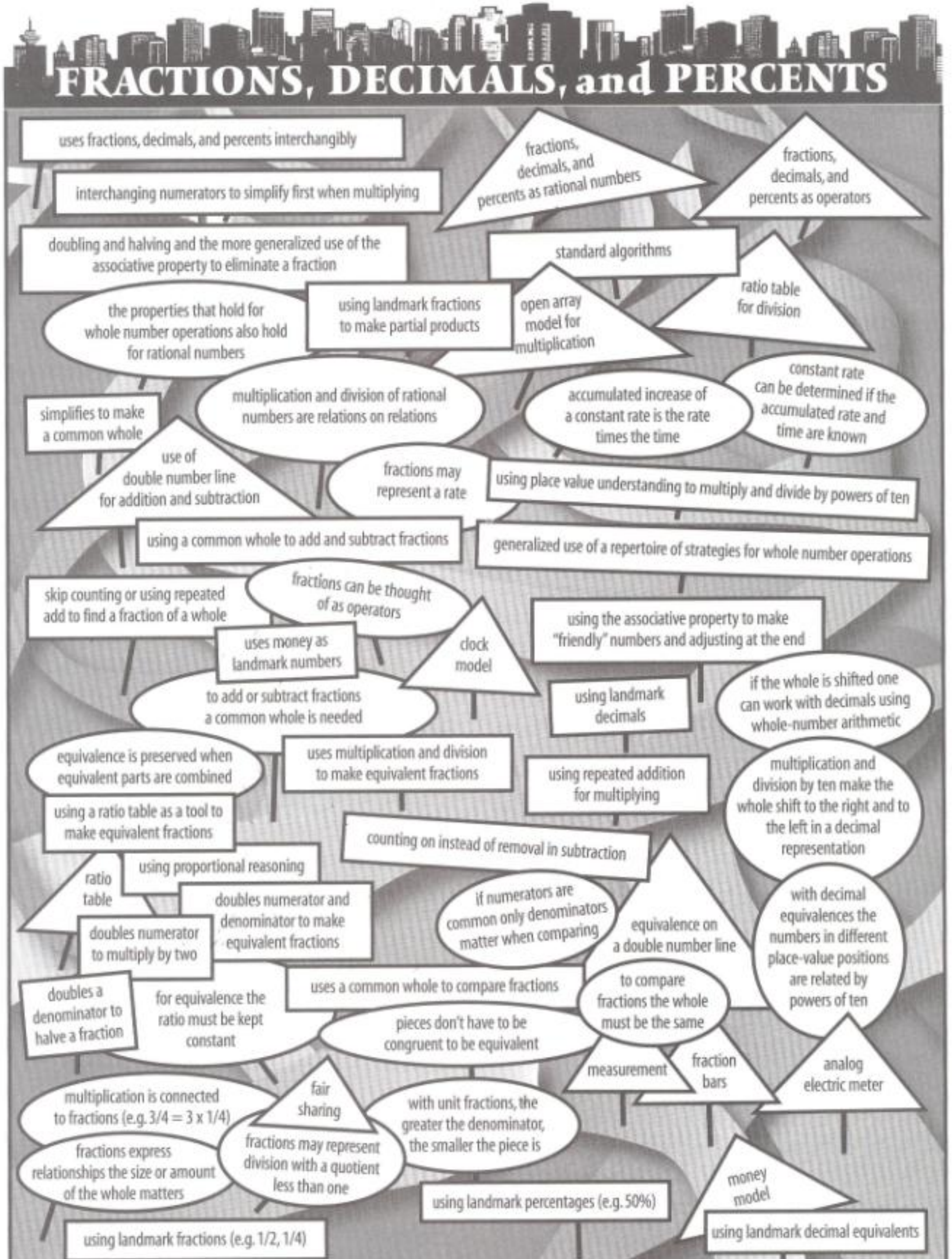


FIG. 1.2. A *knowledge package* for subtraction with regrouping.



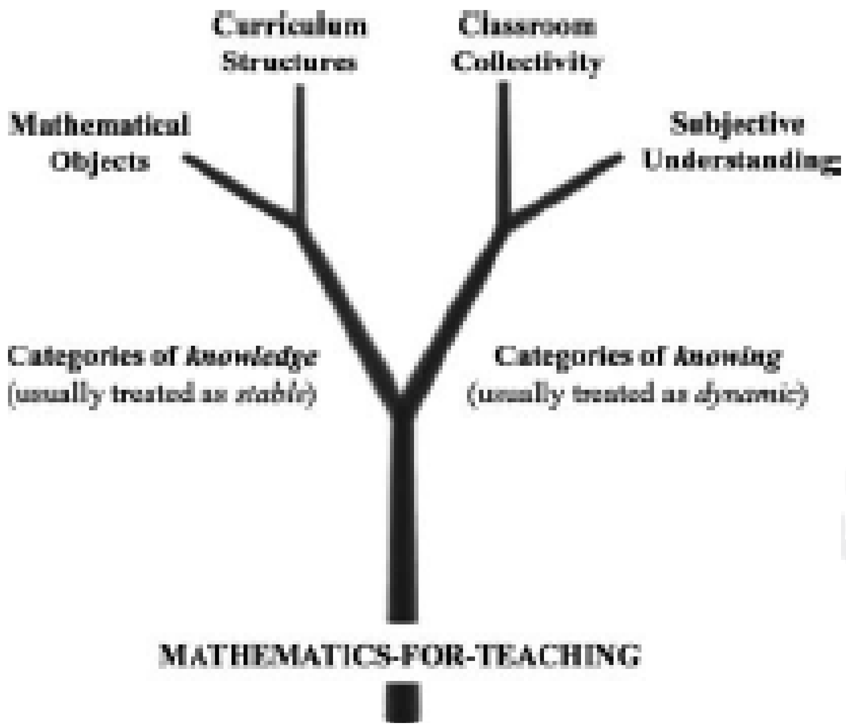
Growing Accessible Interactive Networked Supports

Mathematics Landscape of Learning
(Fosnot, 2003)

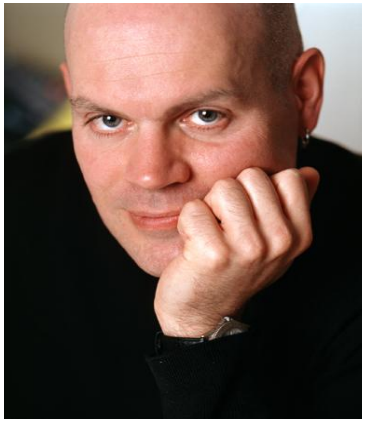
- big ideas or key ideas (ovals)
- strategies (rectangles)
- models of representation (triangles)

(Fosnot, 2002)

Mathematics *for* Teaching
Brent Davis (UBC) and Elaine Simmt (UofA)



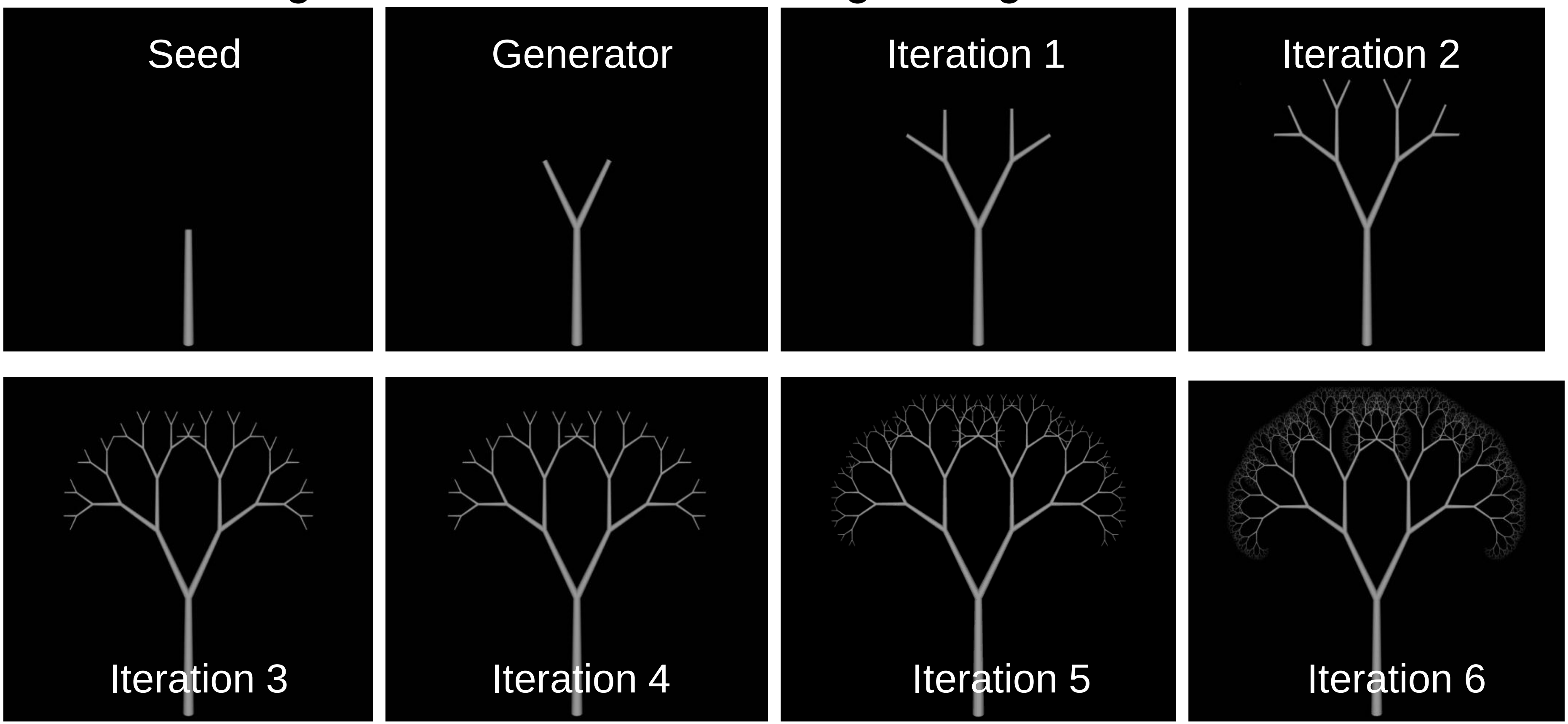
Teachers have embodied, tacit knowledge of mathematics *for teaching.*



Mathematics knowledge is recursively elaborated across the curriculum.

Mathematics Curriculum as a Recursive Elaboration

Visualize Patterning and Algebra curriculum
from Kindergarten to Grade 10 as a growing fractal tree.



Curriculum – Recursively Elaborate

P & A - K to Grade 2

By the end of Kindergarten, children will:

explore, recognize, describe, and create patterns, using a variety of materials in different contexts;

Seed

By the end of Grade 1, students will:

- identify, describe, extend, and create repeating patterns;
- demonstrate an understanding of the concept of equality, using concrete materials and addition and subtraction to 10.

Generator

By the end of Grade 2, students will:

- identify, describe, extend, and create repeating patterns, growing patterns, and shrinking patterns;
- demonstrate an understanding of the concept of equality between pairs of expressions, using concrete materials, symbols, and addition and subtraction to 18.

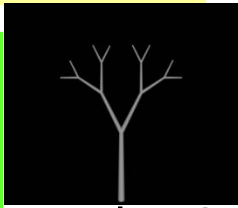
Iteration 1

Curriculum – Recursively Elaborate

P & A - Grades 3 to 5 Expectations

By the end of Grade 3, students will:

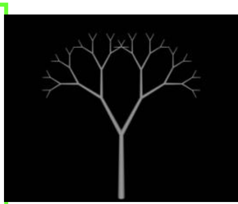
- describe, extend, and create a variety of numeric patterns and geometric patterns;
- demonstrate an understanding of equality between pairs of expressions, using addition and subtraction of one- and two-digit numbers.



Iteration 2

By the end of Grade 4, students will:

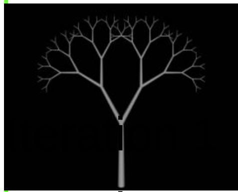
- describe, extend, and create a variety of numeric and geometric patterns, make predictions related to the patterns, and investigate repeating patterns involving reflections;
- demonstrate an understanding of equality between pairs of expressions, using addition, subtraction, and multiplication.



Iteration 3

By the end of Grade 5, students will:

- determine, through investigation using a table of values, relationships in growing and shrinking patterns, and investigate repeating patterns involving translations;
- demonstrate, through investigation, an understanding of the use of variables in equations.



Iteration 4

GAINS M

Curriculum – Recursively Elaborate

P & A Grades 6 to 8 Expectations

By the end of Grade 6, students will:

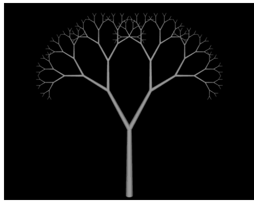
- describe and represent relationships in growing and shrinking patterns (where the terms are whole numbers), and investigate repeating patterns involving rotations;
- use variables in simple algebraic expressions and equations to describe relationships.

By the end of Grade 7, students will:

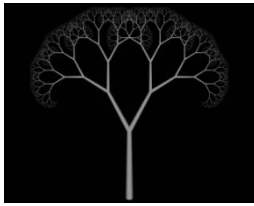
- represent linear growing patterns (where the terms are whole numbers) using concrete materials, graphs, and algebraic expressions;
- model real-life linear relationships graphically and algebraically, and solve simple algebraic equations using a variety of strategies, including inspection and guess and check.

By the end of Grade 8, students will:

- represent linear growing patterns (where the terms are whole numbers) using graphs, algebraic expressions, and equations;
- model linear relationships graphically and algebraically, and solve and verify algebraic equations, using a variety of strategies, including inspection, guess and check, and using a "balance" model.



Iteration 5

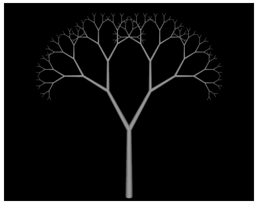


Iteration 6

Learning *Mathematics for Teaching*

Curriculum – Recursively Elaborate

P & A Grades 9 to 10 Expectations



Grade 9

Iteration 7

Grade 10

Iteration 8

determine values of a linear relation by using a table of values, by using the equation of the relation, and by interpolating or extrapolating from the graph of the relation (*Sample problem:* The equation

determine other representations of a linear relation arising from a realistic situation, given one representation (e.g., given a numeric model, determine a graphical model and an algebraic model; given a graph, determine some points on the graph and determine an algebraic model);

construct tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools (e.g., spreadsheets, graphing software, graphing calculators, paper and pencil), for linearly related and non-linearly related data collected from a variety of sources (e.g., experiments, electronic secondary sources, patterning with

determine graphically the point of intersection of two linear relations, and interpret the intersection point in the context of an application (*Sample problem:*

Three-Part Problem Solving Lesson Design

After (Consolidation) – Bus Problem

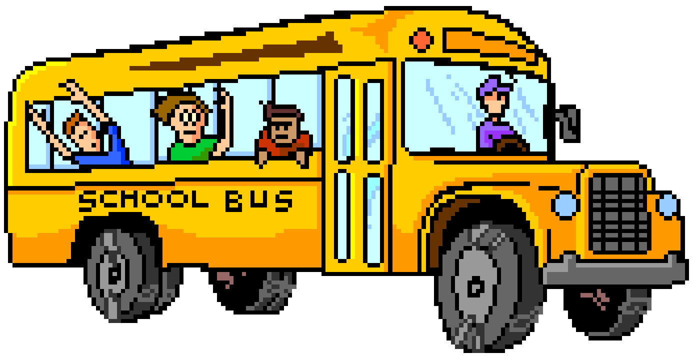
There are 36 children on school bus.

There are 8 more boys than girls.

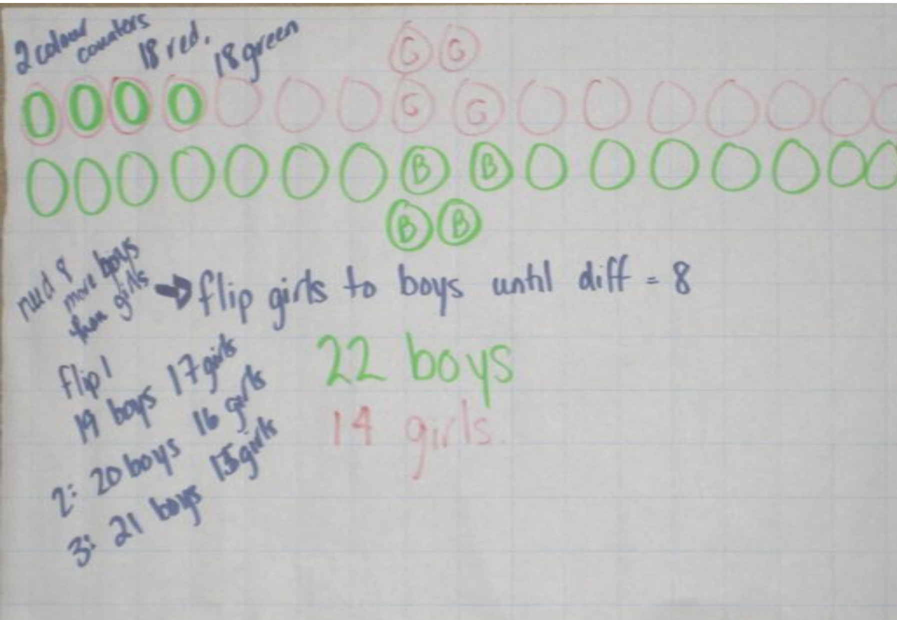
How many boys? How many girls?

a) Solve this problem in 2 different ways.

b) Show your work.

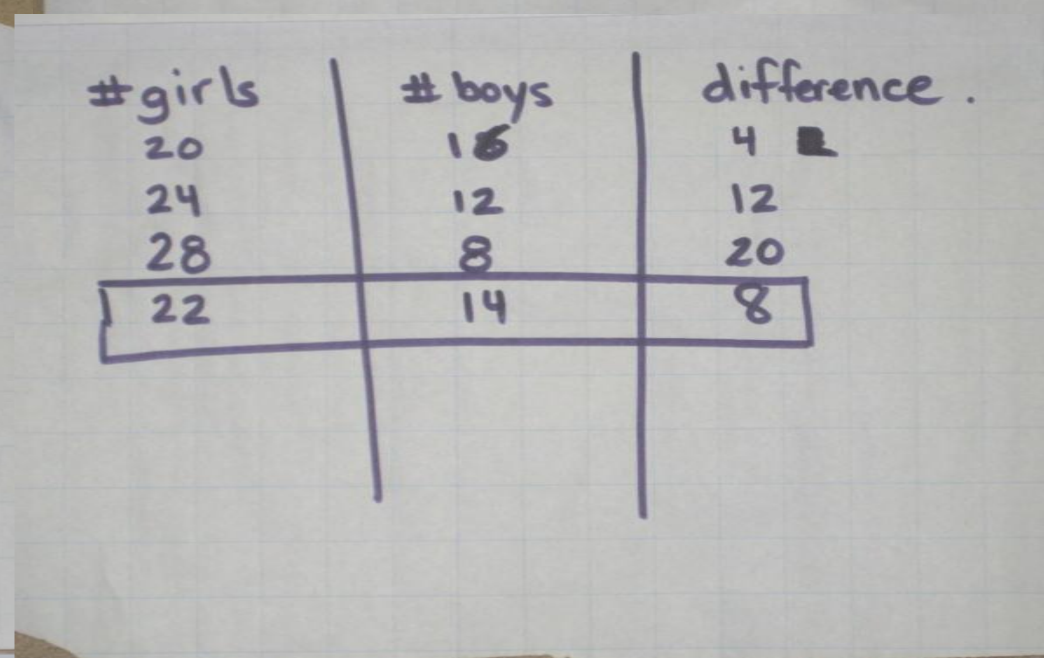


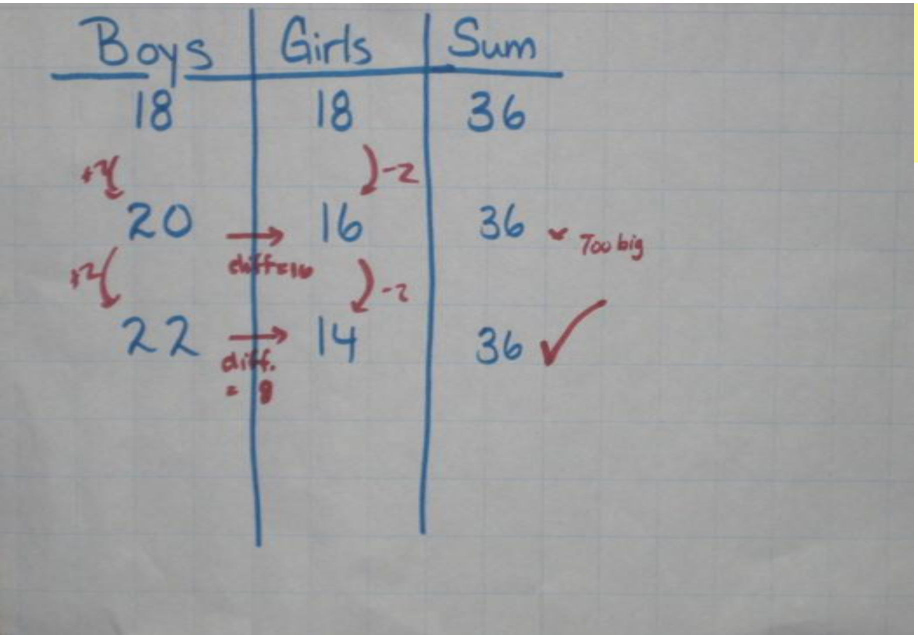
- How are our solutions similar and different?
- What mathematics (curriculum expectations) are demonstrated in the solutions?
- What are the mathematical relationships between the solutions?

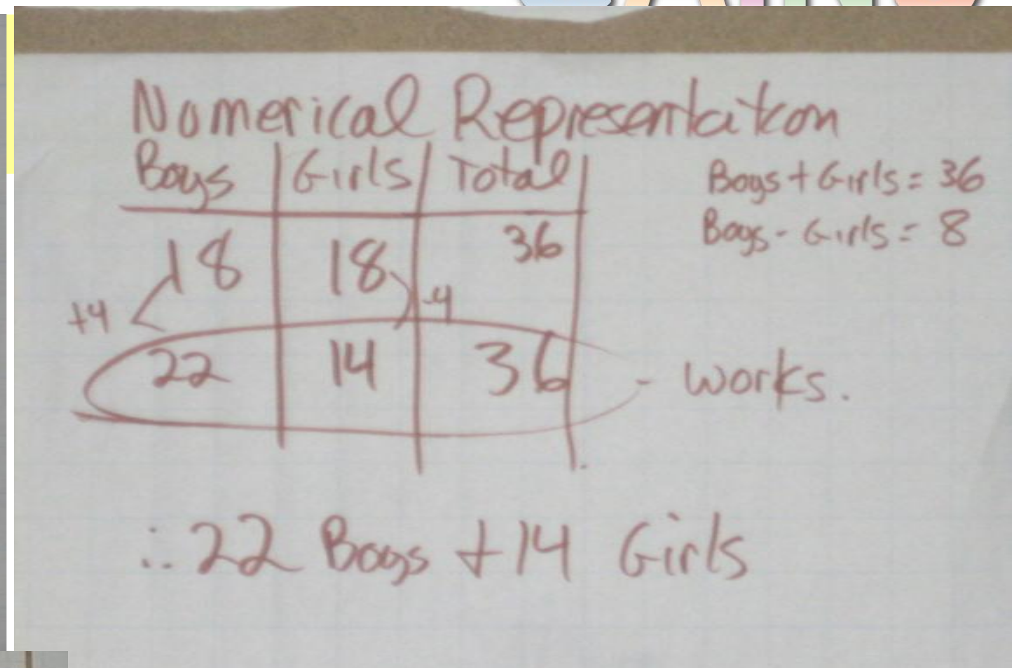

2 coloured counters 18 red, 18 green
need 8 more boys than girls
flip girls to boys until diff = 8
flip 1 19 boys 17 girls
2: 20 boys 16 girls
3: 21 boys 15 girls
22 boys
14 girls

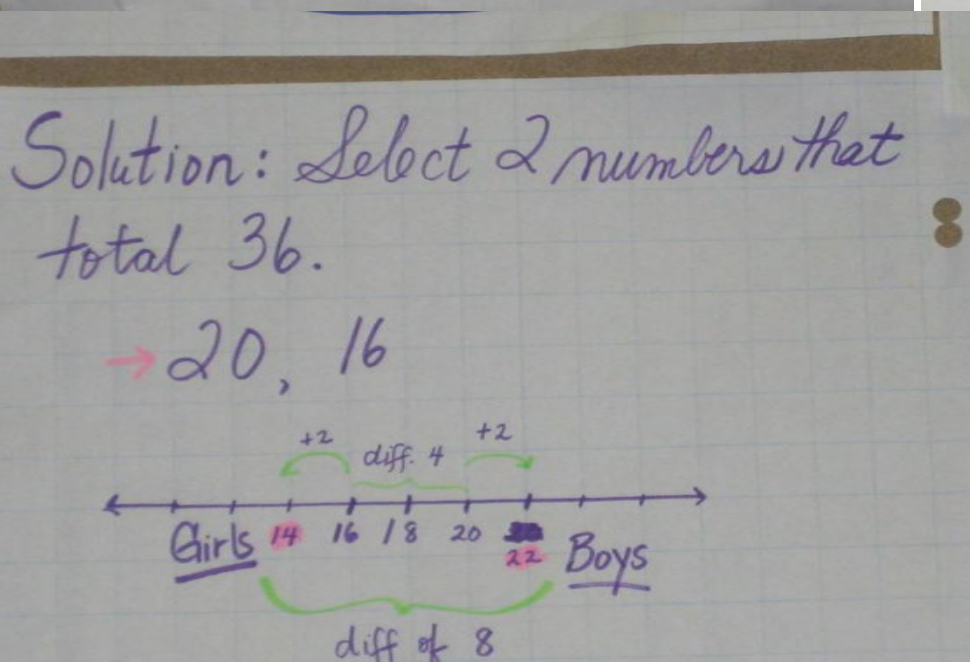

Boys
Girls
eight more boys than girls
22
14

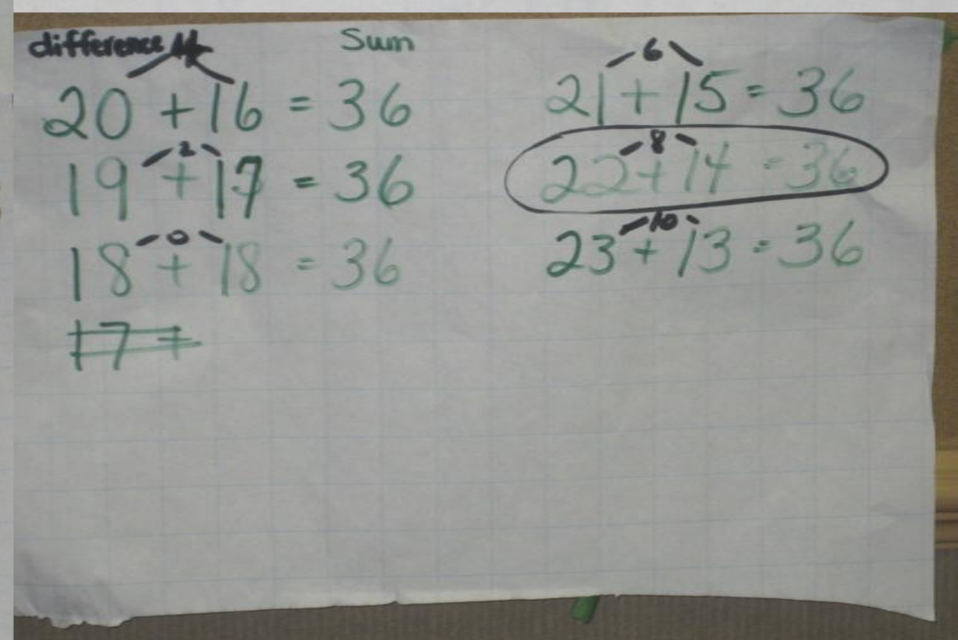

Boys Girls Total More boys
18 18 36 No
19 17 36 2+
20 16 36 4+
21 15 36 6+
22 14 36 8✓


#girls #boys difference
20 16 4
24 12 12
28 8 20
22 14 8

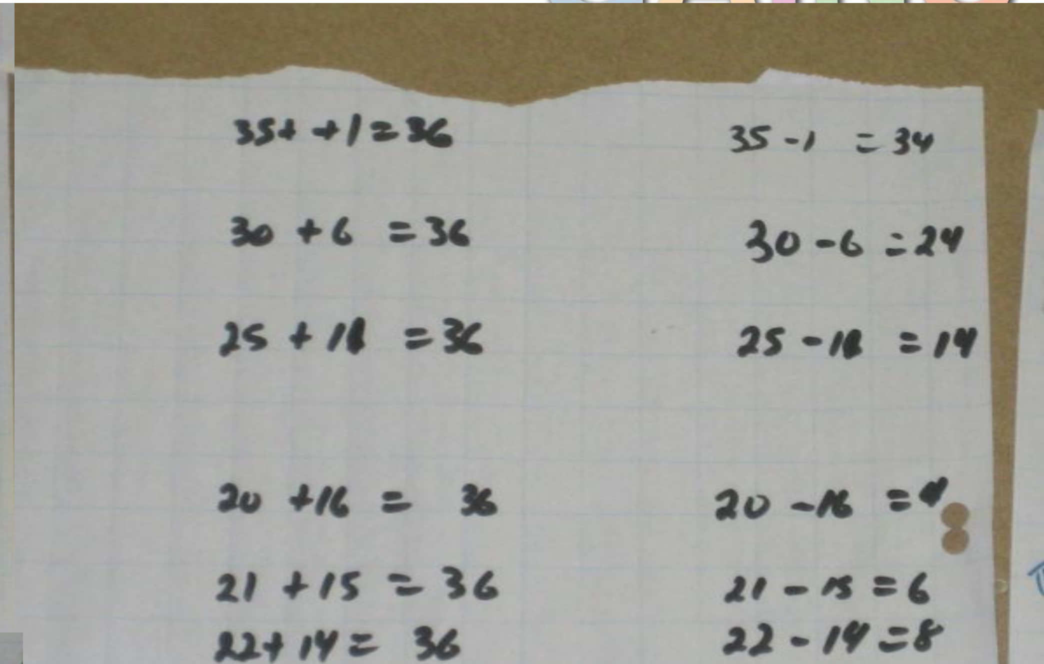

Boys | Girls | Sum
18 18 36
+2 -2
20 16 36 Too big
diff=4
+2 -2
22 14 36
diff. = 8

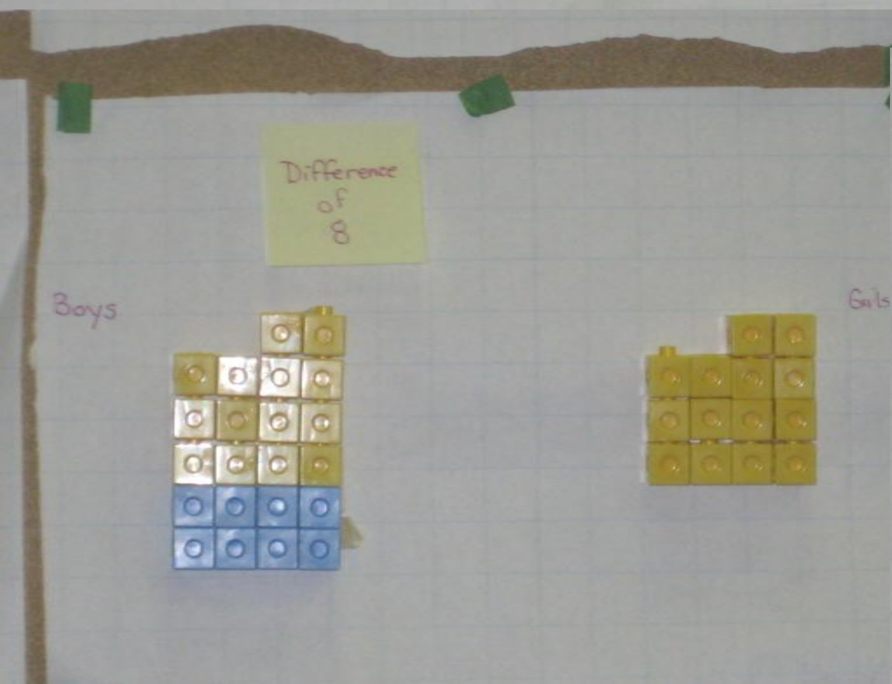

Numerical Representation
Boys | Girls | Total
18 18 36
+4 -4
22 14 36 works.
Boys + Girls = 36
Boys - Girls = 8
∴ 22 Boys + 14 Girls

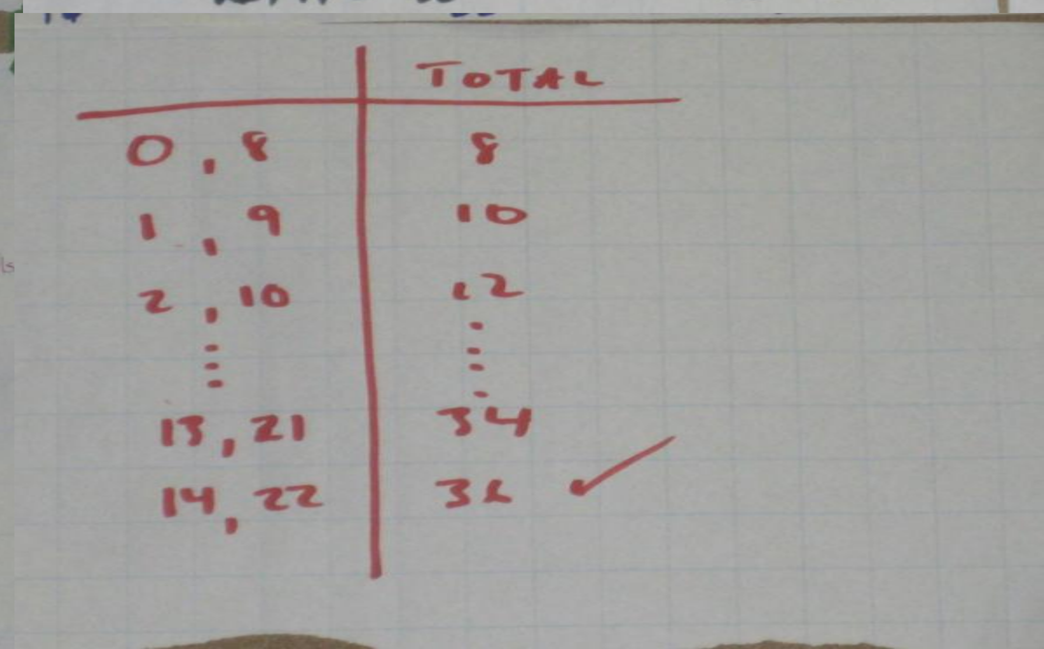

Solution: Select 2 numbers that total 36.
→ 20, 16
+2 diff. 4 +2
Girls 14 16 18 20 22 Boys
diff of 8


difference Sum
20 + 16 = 36
19 + 17 = 36
18 + 18 = 36
21 + 15 = 36
22 + 14 = 36
23 + 13 = 36


SUM = 36
DIFFERENCE
BOYS
GIRLS
18 18 0
19 17 2
20 16 4
21 15 6
22 14 8


35 + 1 = 36
35 - 1 = 34
30 + 6 = 36
30 - 6 = 24
25 + 11 = 36
25 - 11 = 14
20 + 16 = 36
20 - 16 = 4
21 + 15 = 36
21 - 15 = 6
22 + 14 = 36
22 - 14 = 8


Difference of 8
Boys
Girls


TOTAL
0, 8 8
1, 9 10
2, 10 12
13, 21 34
14, 22 36

= boy

= girl

∴ 22 boys + 14 girls

36 students ÷ 2 groups = 18 average in each

$8 \div 2 = 4$ (distance on each side of 18 to move)

$18 + 4 = 22$ boys

$18 - 4 = 14$ girls

① $36 \div 2 = 18$ $8 \div 2 = 4$

$18 + 4 = 22$ $18 - 4 = 14$

① $36 = 2x + 8$

$\quad -8 \qquad -8$

$\frac{28}{2} = \frac{2x}{2}$ $x = 14$

Where x represents # of gir

Let x be the number of girls, and '$x+8$' be the number of boys.

$x + (x+8) = 36$

$2x + 8 = 36$

$2x = 36$

$x = 14$ $x+8 = 14+8 = 22$

∴ There are 14 girls and 22 boys.

Solution:

Let x represent the # of boys.

$\therefore x-8$ represents the # of girls

So, $x+(x-8)=36$

$$2x-8=36$$
$$2x=44$$
$$x=\boxed{22}$$

# of girls $= 22-8 = \boxed{14}$

$$x+y=36$$
$$x-y=8$$
$$2x = 44$$
$$x=22$$

$\therefore y=36-22$

$y=14$

$\therefore$ Boys $=22$

Girls $=14$

② ① $x+y=36$

$+$② $x-y=8$

$$\frac{2x}{2}=\frac{44}{2}$$
$$\boxed{x=22}$$

sub $x=22$ into ②

$22-y=8$

$22-8=y$

$\boxed{14=y}$

$$x+y=36$$
$$x-y=8$$
$$2x=44$$
$$x=22$$

$22-y=8$

$22-8=y$

$14=y$

2 numbers 14, 22

$x-y=8$ ①

$x+y=36$ ②

$2x=44$ ①−②

$x=22$

① $x+y=36$

$22+y=36$

$y=14$

PROOF

$x-y=8$

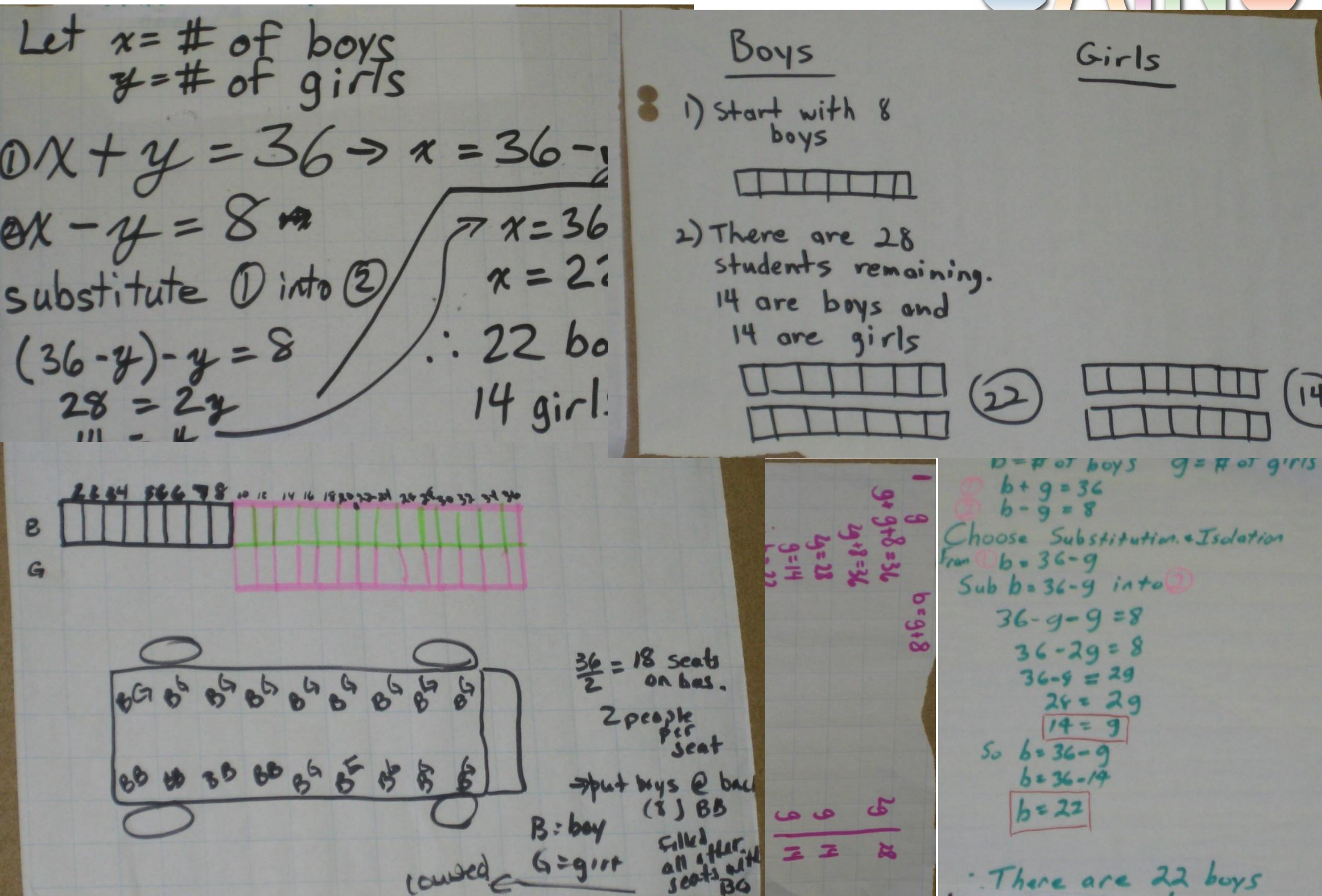
$22-14=8$

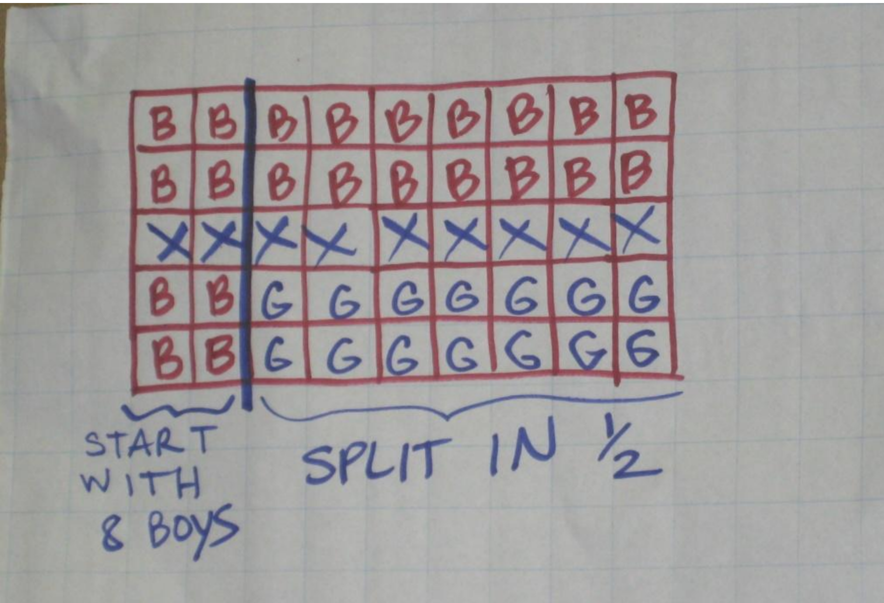
$8=8$

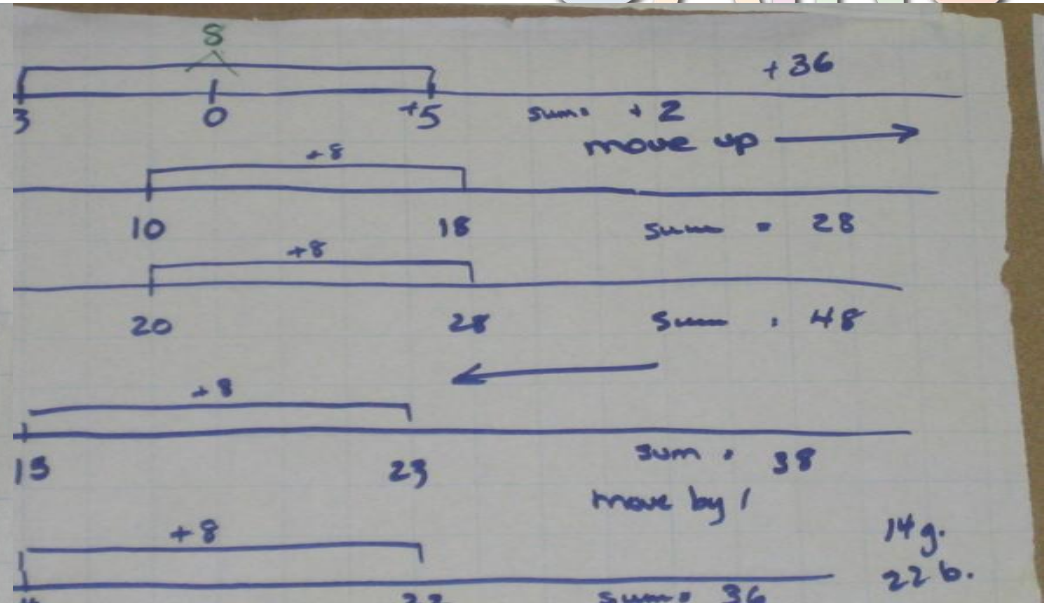
$x=$ # boys

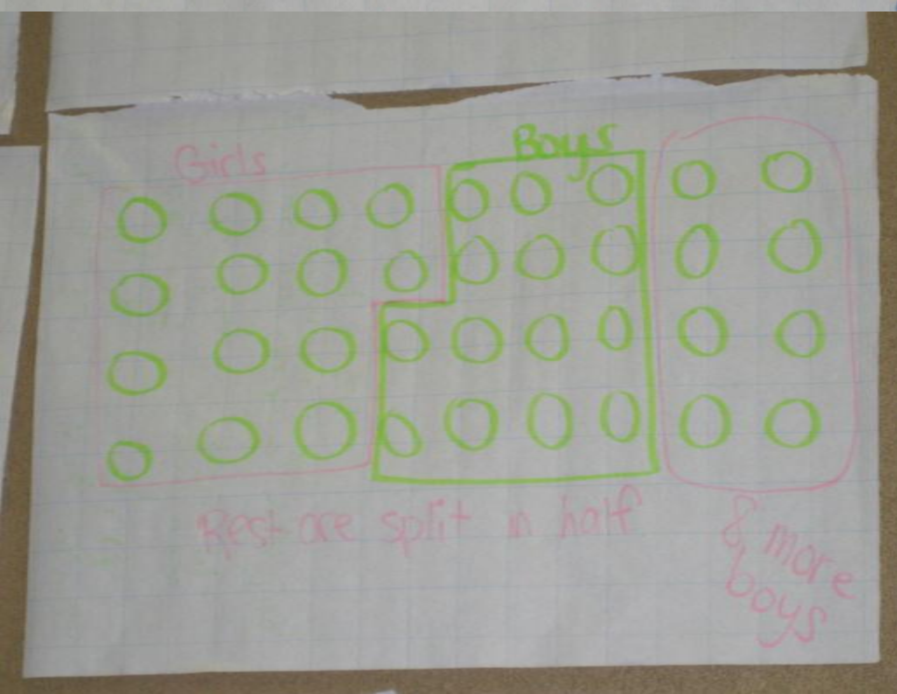
$y=$ # girls

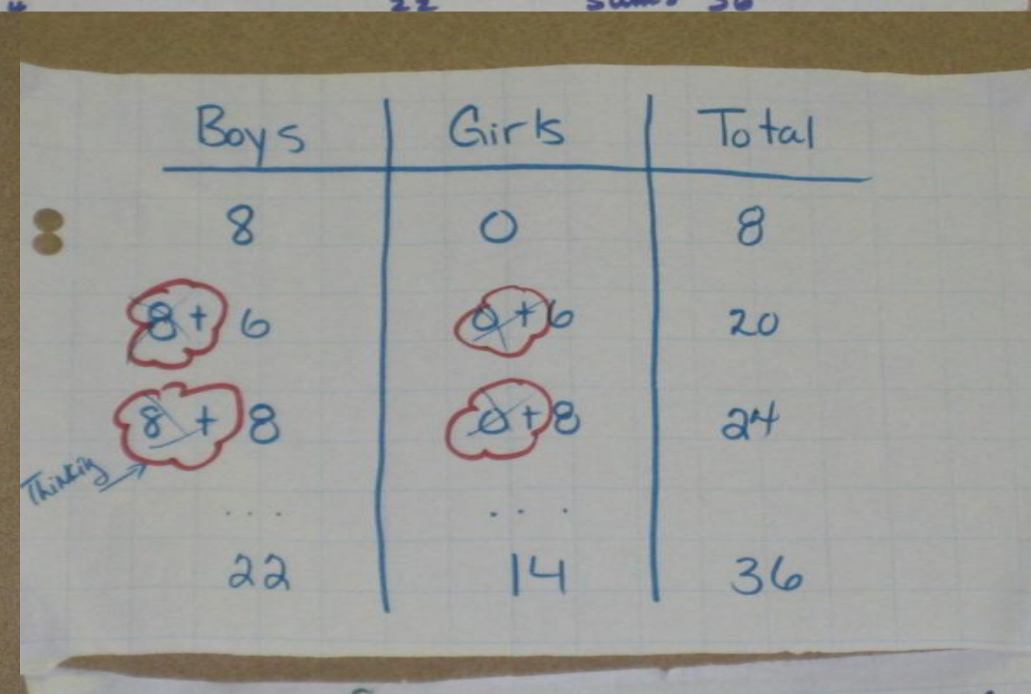
GAINS


Let x = # of boys
y = # of girls
① x + y = 36 → x = 36 -
② x - y = 8
substitute ① into ②
(36 - y) - y = 8
28 = 2y
x = 36
x = 2
∴ 22 bo
14 girl
Boys
Girls
1) Start with 8 boys
2) There are 28 students remaining.
14 are boys and
14 are girls
22
14
B
G
36/2 = 18 seats on bus.
2 people per seat
put boys @ back
(8) BB
filled all other seats with BG
B: boy
G = girl
b = # of boys
g = # of girls
b + g = 36
b - g = 8
Choose Substitution + Isolation
From ① b = 36 - g
Sub b = 36 - g into ②
36 - g - g = 8
36 - 2g = 8
36 - 8 = 2g
28 = 2g
14 = g
So b = 36 - g
b = 36 - 14
b = 22
∴ There are 22 boys


START WITH 8 BOYS
SPLIT IN ½

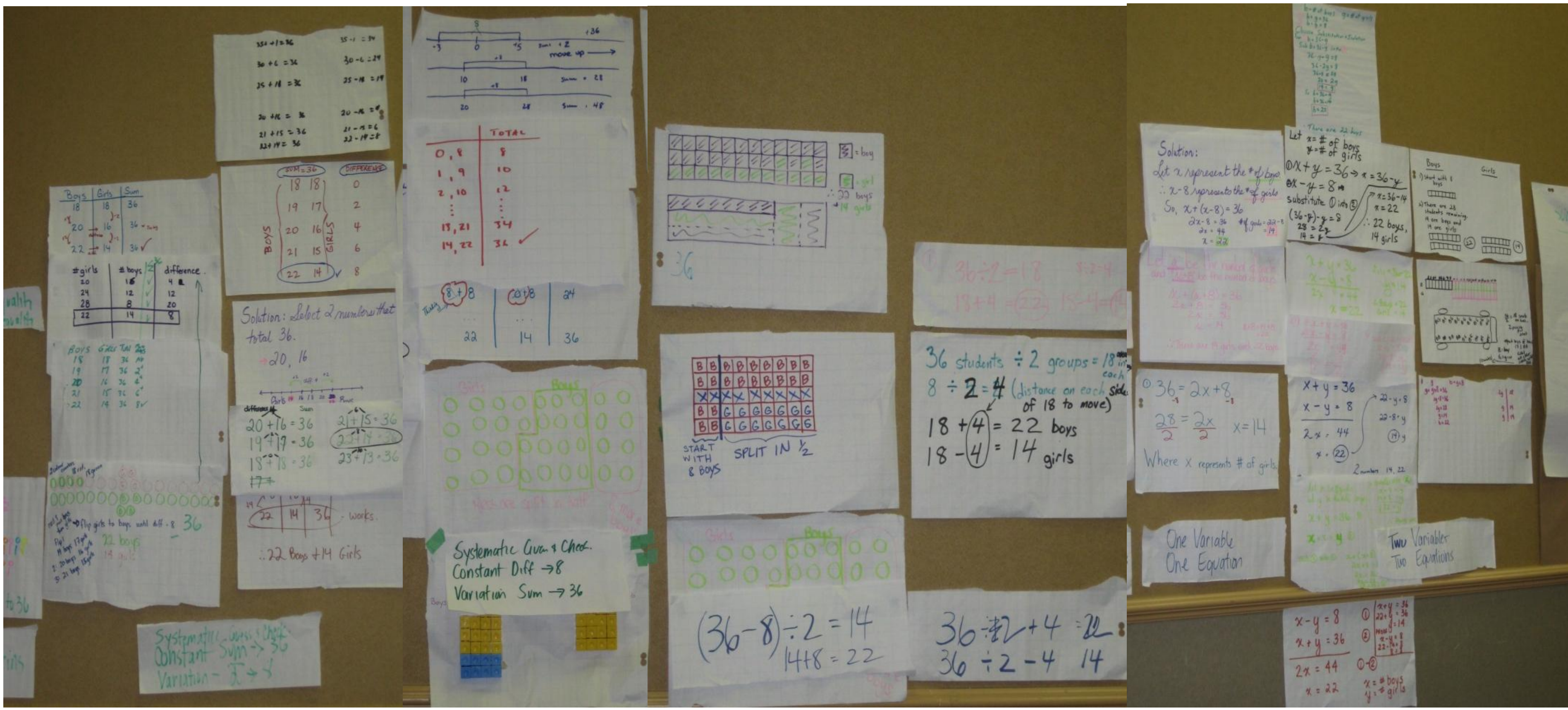

+36
sum = +2
move up
sum = 28
sum = 48
sum = 38
move by 1
14 g.
22 b.
sum = 36


Girls
Boys
Rest are split in half
& more boys


Boys
Girls
Total
Thinking

Bus Problem – Partial Bansho
(Also includes BDA)

Arithmetic to Algebra



Systematic Guess and Check → (Constant Sum, Variation Difference) (Constant Diff, Variation, Sum)

Order Of Operations→

1 and 2 Variable Equations (Substitution, Simplifying (Equation, Isolating Variable)

Three-Part Problem Solving Lesson Design
After (Practice)

- What problem(s) should we provide to enable students to practise (in pairs, individually) what they have learned?
- What learning goals (curriculum expectations) does your Practice Problem?

There are 36 children on school bus. There are 8 more boys than girls.
How many boys?
How many girls?

a) Solve this problem in 2 different ways.
b) Show your work.

Larger numbers to discourage some numeric strategies and to encourage strategies that are possibly generalizable

Developing a Math Talk Community

- Never Say Anything a Kid Can Say

Never Say Anything a Kid Can Say!

STEVEN C. REINHART

AFTER EXTENSIVE PLANNING, I PRESENTED what should have been a masterpiece lesson. I worked several examples on the overhead projector, answered every student's question in great detail, and explained the concept so clearly that surely my students understood. The next day, however, it became obvious that the students were totally confused. In my early years of teaching, this situa-

Before long, I noticed that the familiar teacher-centered, direct-instruction model often did not fit well with the more in-depth problems and tasks that I was using. The information that I had gathered also suggested teaching in nontraditional ways. It was not enough to teach better mathematics; I also had to teach mathematics better. Making changes in instruction proved difficult because I had to learn to teach in ways that I had never ob-

Read 1 page ... Key idea about developing a math-talk community.

Origins of "Ontario" Bansho

(Yoshida, 2003; Shimizu, 2005)

Components of Japanese Teaching

Blackboard is used to show the flow of the lesson process and to

- keep a record of the lesson to help students remember what they need to do and think
- help student see connections and progression of parts of the lesson
- contrast and discuss ideas students presented from their solutions
- organize student thinking and develop new ideas

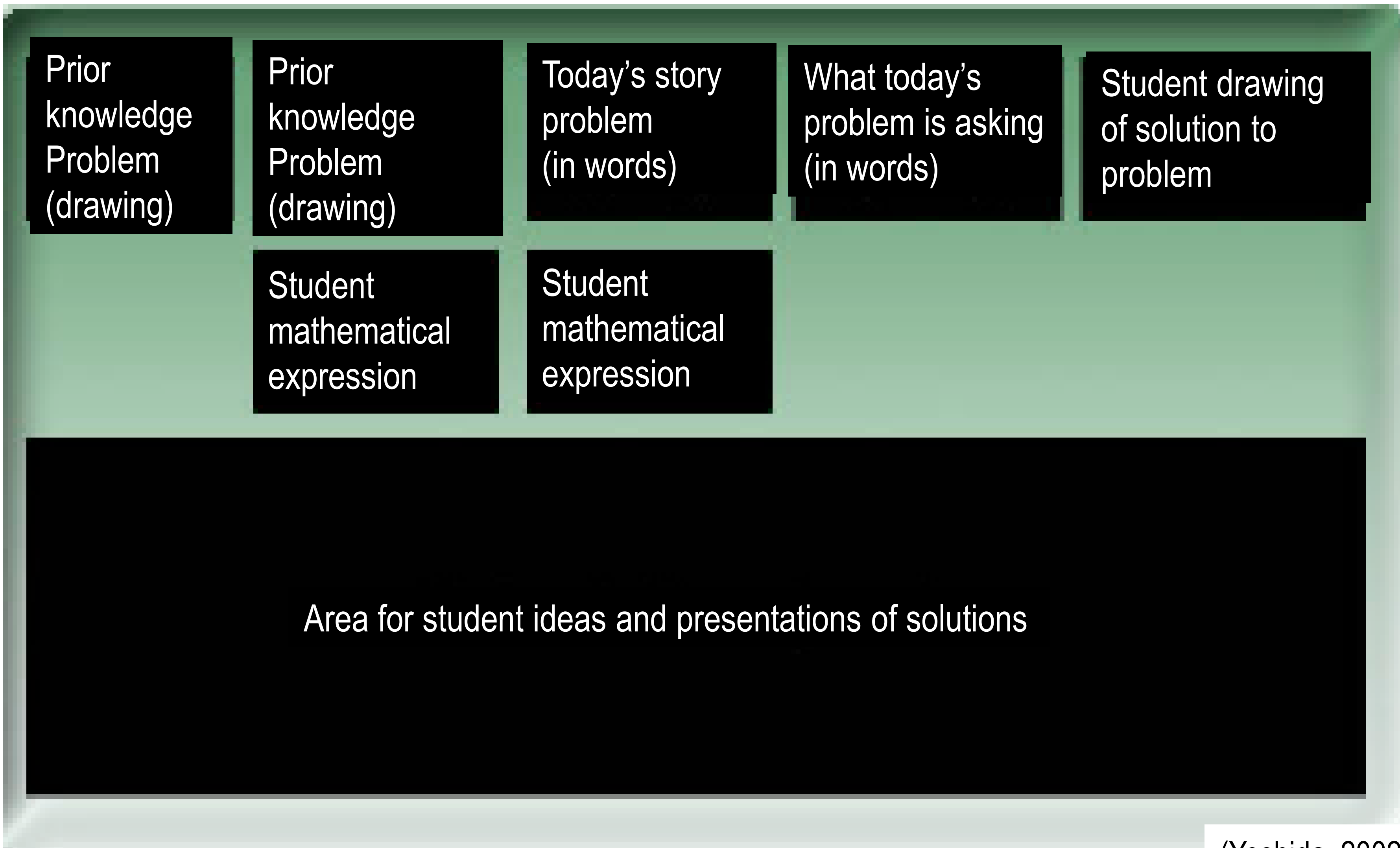
Bansho – means board writing – technical term created by Japanese teachers; rarely erase what they write on the blackboard … all recordings have a mathematical meaning and purpose

Hatsumon – asking a question to stimulate students' thinking

Neriage – teacher coordinating student sharing, analysis, and discussion

Matome – teacher summarizing key mathematical ideas from the lesson

Origins of "Ontario" Bansho
- A Sample Japanese Blackboard Plan



(Yoshida, 2002)